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ANOMALOUS TRANSPORT IN PLASMA
(Summary of Ph.D. thesis)

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An important issue in the study of the controlled nuclear fusion is the confinement of plasma by means of a magnetic field for a time, which is sufficient for the start of the nuclear fusion reactions. As a consequence, the understanding of the particle and energy cross-field fluxes is crucial for the control of unfavorable losses in fusion devices such as a tokamak.

It is well known that the results obtained by the "classical" approach are quite different when compared to those obtained by experiments in specific devices (tokamak or stellarator): the classical theory grossly underestimates the transport coefficients (e.g. the diffusion coefficients) in fusion plasma, usually by several orders of magnitude. The classical predictions are obtained in the framework of the "***Classical Transport Theory*"** and they are not totally confirmed by experiments. The effect of the magnetic field geometry on the free motion of the particles was included into a new theory. The magnetic field produced in fusion devices has a ***toroidal geometry*** and the effect of the global magnetic configuration has a surprisingly strong effect on the transport properties of the plasma. The classical theory, which takes into account for collisions and also for the magnetic field geometry has been called the "***Neoclassical Transport Theory"***. The transport coefficients obtained in the framework of the latter theory are strongly enhanced when compared to the classical ones and the results obtained using the tools of this theory, in some special situations, are in agreement with the experimental data. But, even the neoclassical enhancement is still far from explaining the large effects observed in tokamaks. There are a lot of numbers of physical processes or experimental results that cannot be explained by ***Neoclassical Transport Theory***. Taking into consideration the long range of the Coulomb forces and the collective nature of the plasma which dominates its behavior a new qualitative and quantitative theory has been developed: "***Anomalous Transport Theory"***. Due to the collective phenomena some organized movements in plasma appear. These waves" (or "modes") which may propagate, grow or decay in time can interact with the individual particles and among themselves. The plasma can evolve towards a turbulent state (i.e., a state far away from a steady state). The study of these non-equilibrium processes is the object of the ***Anomalous Transport Theory***. In principle, the complicated processes from inside of the turbulent plasma are still described by the laws of statistical mechanics and their evolution is contained in the Liouville equation.

In all transport theories, classical, neoclassical or anomalous, we are interested especially in the calculation of the diffusion coefficients; this goal is achieved if we take into consideration the combined effect of the collisions and of the level of turbulence. The turbulent state of plasma can be described in a statistical manner applying the specific methods of the statistical mechanics adapted to the current problem. Usually, an ensemble of realizations of the fluctuating quantity is built. The latter can be an electrostatic potential or a magnetic field potential for which the statistical properties are specified *a priori* with an *a posteriori* validation. In this framework the distribution function is a stochastic quantity instead of a deterministic one as in the ordinary kinetic theory. The development of the anomalous transport based directly on a kinetic equation is very difficult. The analysis begins from the equations of motion (in general of a Langevin type) of a test particle moving in a stochastic velocity field. In fusion plasma there are two different turbulent states. One of them is related to the electrostatic turbulence, which is associated with small frequencies instabilities, which in turn generates an electric turbulent field. This field enhances the growth of the electrostatic drift and in consequence a perturbation of the radial velocity appears and losses in the radial direction are generated. This is equivalent with a generation of the anomalous radial transport. The drift modes always exist because they are generated by the density and/or temperature gradients specific to plasma fusion.

The second type of turbulence in plasma fusion is the magnetic one, which is generated by the internal instabilities that destroy the nested magnetic surfaces and the particles, which usually follow the magnetic field lines, will be lost in the radial direction. Again, an anomalous transport is generated.

In our thesis we have used stochastic models in order to study the anomalous transport properties of the fusion plasma.

In this Ph.D. thesis it were studied mostly the diffusion processes, for various configurations of stochastic fields (magnetic and/or electrostatic) that are typical for fusion plasma. In particular, we have studied the diffusion of stochastic magnetic field lines, the diffusion of an electron in a combined structure of stochastic magnetic and electrostatic fields and the zonal flow generation (the quasi-particles diffusion) in the drift electrostatic turbulence. In all these models it were solved numerically the differential stochastic Langevin equations associated to the system. The final goal for each individual study was the estimation of the diffusion tensor components.

In **Chapter 2** the theory of differential stochastic Langevin equations, which are the characteristic equations of hybrid kinetic equation (HKE) was shortly presented (Paragraph 2.1). In this chapter, the HKE concept was also presented (Paragraph 2.2). The both approaches are very used for the study of the stochastic processes typical for fusion plasma. There are a lot of scientific works that are based on these two approaches. In the **References** of this thesis the most significant papers are mentioned.

In **Chapter 3,** a new method devoted especially for the evaluation of the diffusion coefficients in the case of a relatively strong turbulence regime, namely the **Decorrelation Trajectory Method (DCT)**, is shortly presented. The latter is a semi-analytical method and is valid for both weak and relatively strong turbulence regimes. For all the models studied in this thesis (Chapters 4-6) the DCT method was used.

In **Chapter 4** the decorrelation trajectory method is applied to the diffusion of magnetic field lines of a perturbed sheared slab magnetic field. Some decorrelation trajectories for several values of the magnetic Kubo number K_m and of the shear parameter are exhibited. The asymmetry of the decorrelation trajectories appears in comparison with those obtained in the purely electrostatic case studied in earlier work. The running and the asymptotic diffusion tensor components are calculated and displayed. We restricted ourselves to the study of the geometrical aspect of the problem: we thus analyzed the properties of the magnetic lines of a sheared stochastic magnetic field alone. We considered the simplified model of the slab approximation including the magnetic shear. The presence of the shear influences the length of a magnetic line between two-fixed z -values (z is the coordinate in the direction of the reference magnetic field and plays the role of time). It was shown that the magnetic shear and the magnetic Kubo number K_m have an important influence on the diffusion of the magnetic field lines. The problem is simplified whenever the study is restricted to the stochastic magnetic fields with small amplitudes and/or large perpendicular correlation lengths for which the magnetic Kubo number is small. The main contribution of our work consists, however, in analyzing the influence of the shear in both weak and strong turbulence regimes for the stochastic magnetic fields. Until now the influence of the shear on the diffusion coefficients was analyzed only in the quasilinear limit, i.e. the relatively small Kubo number regime. The diffusion coefficients for the magnetic field lines were studied for both small and high Kubo number regimes in many papers but only for the shearless case.

Our analysis is not restricted only in a range of small magnetic Kubo number $K_m<1$. Using the recently developed method of investigation of the diffusion in a stochastic velocity field, namely DCT method, we extended our analysis to a relatively high Kubo number regime $K_m>1$. Because of the existence of the two parameters, the magnetic Kubo number K_m and the shear parameter θ_s, a richer class of behaviors of the diffusion coefficients was observed. The competition between these two parameters plays an important role and seems to be decisive in the determination of the trapping effects.

The diffusion of an electron in a combined two-dimensional fluctuating electrostatic field and a sheared slab magnetic geometry is considered in **Chapter 5**. The model used for the computations is similar to a Langevin type system of equations that describes the motion of a guiding center through the highest significant order of approximation in the drift parameter. The statistical characteristics of the stochastic electric field are based on experimental measurements. The stochastic parallel motion along the magnetic field is neglected so that the motion is considered two-dimensional. Using the decorrelation trajectory method we studied the effect of the shear on the trapping, i.e. the competition between the shear Kubo number K_s and the electrostatic Kubo number K on the shape of the decorrelation trajectory. We also considered the decorrelation trajectories, the running and the asymptotic diffusion coefficients, as functions of the above mentioned parameters. Here, we analyzed in detail the influence of the magnetic shear on the "radial" and "poloidal" diffusion of a thermal electron in both weak and relatively strong turbulence regimes for the stochastic electrostatic fields. The problem of the motion of the guiding center in a sheared slab in the presence of a magnetic island and of a single electrostatic wave was previously analyzed by direct numerical simulation in some plasma works. However, to the best of our knowledge, the influence of the magnetic shear on the diffusion coefficients of an electron moving in an electrostatic fluctuating field was not studied for both small and high Kubo number regimes using the decorrelation trajectory method.

A comparison between the diffusion coefficients obtained by DCT and by direct numerical simulation (on a parallel computer) was also done; a first confirmation of the validity of DCT for this kind of Langevin system of equations was obtained.

In **Chapter 6** we analyzed the running and asymptotic diffusion coefficients of plasma in the case of zonal flow generation by an anisotropic stochastic electrostatic potential. Both the weak and relatively strong turbulence regimes were analyzed. The analysis of the diffusion coefficients in wave vector space provides an illustration of the fragmentation of drift wave structures in the radial direction and the generation of long-wavelength structures in the poloidal direction that are identified as zonal flows. We have shown that the fragmentation of drift wave structures is strongly influenced by the anisotropy parameter, the electrostatic Kubo number, the diamagnetic Kubo number and by the initial values of the wave vector.

A zonal flow generation analysis that takes into consideration the combined nonlinear influence of the stochastic anisotropy and Kubo numbers has an original character; this was developed here for the first time.

A complete study of the influence of all parameters (entering our theory) on the zonal flow generation was fulfilled. In order to accomplish this a large number of runs (of a special parallel code) were performed and analyzed in order to obtain a clear overview about the appearance of the zonal flow effect.

The final conclusions are presented in **Chapter 7**.

In **Appendix** are presented briefly the information about the original numerical codes that have been used for the numerical calculations of the diffusion tensors.

The original results included in thesis were published or submitted for publication in the most significant international journal devoted to plasma physics.

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