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Summary of Ph.D. Thesis

TOPOLOGICAL BF THEORIES

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Contents

SUMMARY

Topological field theories originate in the papers of Schwarz and Witten. Initially, Schwarz shown that one of the topological invariants, namely the Ray-Singer torsion, can be represented as the partition function of a certain quantum field theory. Subsequently, Witten constructed a framework for understanding Morse theory in terms of supersymmetric quantum mechanics. These two constructions represent the prototypes of all topological field theories. The model used by Witten has been applied to classical index theorems and, moreover, suggested some generalizations that led to new mathematical results on holomorphic Morse inequalities. Starting with these results, further developments in the domain of topological field theories have been achieved. The Becchi-Rouet-Stora-Tyutin (BRST) symmetry allowed for a new definition of topological field theories as theories whose BRST-invariant Hamiltonian is also BRST-exact.

An important class of topological theories of Schwarz type is the class of BF models. This type of models describes three-dimensional quantum gravity and is useful at the study of four-dimensional quantum gravity in Ashtekar–Rovelli–Smolin formulation. Two-dimensional BF models are correlated to Poisson sigma models from various two-dimensional gravities. The analysis of Poisson sigma models — including their relationship to two-dimensional gravity and the study of classical solutions — has been intensively studied in the literature.

In this thesis we approach the problem of construction of some classes of interacting BF models in the context of the BRST formalism. In view of this, we use the method of the deformation of the BRST charge and BRST-invariant Hamiltonian. Both methods rely on specific techniques of local BRST cohomology. The main hypotheses in which we construct the above mentioned interactions are: space-time locality, Poincare invariance, smoothness of deformations in the coupling constant and the preservation of the number of derivatives on each field. The first two hypotheses implies that the resulting interacting theory must be local in space-time and Poincare invariant. The smoothness of deformations means that the deformed objects that contribute to the construction of interactions must be smooth in the coupling constant and reduce to the objects corresponding to the free theory in the zero limit of the coupling constant. The preservation of the number of derivatives on each field implies two aspects that must be simultaneously fulfilled: (i) the differential order of each free field equation must coincide

with that of the corresponding interacting field equation; (ii) the maximum number of space-time derivatives from the interacting vertices cannot exceed the maximum number of derivatives from the free Lagrangian.

The main results obtained can be synthesized into:

- obtaining self-interactions for certain classes of BF models;
- generation of couplings between some classes of BF theories and matter theories;
- construction of interactions between a class of BF models and a system of massless vector fields.

These results are reported in papers [1]-[8].
The thesis is organized in eight chapters. Chapter 1 is introductory.

Chapter 2 exposes the construction of consistent interactions in the BRST formalism. It is well known that the BRST symmetry admits a Lagrangian and a Hamiltonian version. In both situations the BRST symmetry acts as a canonically generated differential: by the solution to the master equation at the Lagrangian level and by the BRST charge in the Hamiltonian formulation. In this sense, it has been proved that one can reformulate the classical problem of construction of consistent interactions as a problem of the deformation of the solution to the master equation (at the Lagrangian level) or as a problem of the simultaneous deformation of the BRST charge and of the BRST-invariant Hamiltonian (at the Hamiltonian level). It exhibits introductory, monographic features. The next chapters expose the original contributions of the author in the domain of the Ph.D. thesis.

Chapter 3 is dedicated to the Hamiltonian formulation of self-interactions for a set of free BF models with the field spectrum consisting in a set of scalar fields, two collections of 1-forms and a system of 2-forms. The resulting interacting model may be interpreted in terms of a Poisson-like structure. The gauge transformations of the interacting theory are non-Abelian and the associated gauge algebra is open. The results from this chapter are published in [1]-[3].

In the sequel there are studied the couplings between certain classes of BF models and matter fields. Thus, Chapter 4 exposes the construction of couplings between a single BF model (with the field spectrum consisting in one scalar field, two 1-forms and a 2-form) and a set of matter fields. The existence of such couplings requires that the matter theory admits a one-parameter rigid symmetry. Under these circumstances, it is shown that at order one in the coupling constant the interaction vertex reads as $U(\varphi)\, j^{\mu} A_{\mu}$, where j^{μ} is the current corresponding to the above mentioned rigid symmetry and $U(\varphi)$ is an arbitrary function depending only on the undifferentiated scalar field. Regarding the higher-order (in the coupling constant) interacting terms, there appear two distinct situations: (a) if the current j^{μ} he gauge version of the rigid symmetry, then there appear no higher-order terms and the function $U(\varphi)$ remains arbitrary; (b) in the opposite situation, there appear at least second-order interactions terms, but the function $U(\varphi)$ may be restricted. The general procedure has been exemplified on the complex scalar field, the Dirac field, and the massive spin-3/2 field. The results from this chapter are published in [4]-[5].

Chapter 5 is dedicated to the generation of interactions between a collection of BF models (with the same field spectrum like in Chapter 3) and matter fields. In this situation the appearance of couplings between the BF sector and matter fields requires that the matter theory must be invariant under some rigid symmetries leading to the conserved currents j_a^{μ}, whose number must be equal to the number of BF fields from the collection. The existence of higher-order deformations (in the coupling constant) requires that the generators of the above mentioned rigid symmetries either commute or generate a Lie algebra $L(\mathcal{G})$. Corresponding to these two cases, the following situations are encountered: (i) if the matter currents are invariant (for commuting rigid generators) or transform according to the adjoint representation of $L(\mathcal{G})$ (for Lie-type rigid generators) under the gauge version of the rigid symmetries, then all the higher-order deformations are trivial; (ii) in the opposite situation, at least the second-order deformation id nontrivial, but in principle it is possible to obtain other nontrivial deformations as well. The general procedure developed in this chapter has been exemplified on a collection of real scalar fields and respectively a set of Dirac fields. The results from this chapter are published in [6].

Next, Chapter 6 approaches the problem of interactions between a system of BF models (with the same field spectrum like in Chapter 3) and a collection of massless vector fields V_{μ}^{A}. In this case there appear interacting terms at both order one and order two in the coupling constant. The emerging couplings include generalized Yang-Mills vertices (of orders three and four) with respect to the vector fields V_{μ}^{A}. The gauge transformations of the interacting theory are deformed with respect to the initial ones, while the algebra of deformed gauge transformations is open. The results from this chapter are published in [7].

Chapter 7 analyzes the problem of constructing self-interactions for a free BF model with a more extended field spectrum, consisting in one scalar field, two types of 1-forms, two sorts of 2-forms and a 3-form. The general form of the interacting Lagrangian has been obtained, as well as the gauge transformations, which turn to be non-Abelian. The associated gauge algebra is open. The results from this chapter are published in [8].

The last chapter includes the main conclusions of the thesis.

Selected references

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